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The Direct Measurement of Damping Coefficients Using An Inverse Eigenvalue Method

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New London, Connecticut**

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PREFACE

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13. ABSTRACT (Maximum 200 words) A direct measurement technique to identify the viscous damping coefficient of one-dimensional devices has been developed. The frequency-dependent properties of the damping coefficient are inherent in this formulation. It has been shown that the variance in the prediction of the damping coefficient is small. The experimental apparatus is extremely easy to build, and the raw data necessary to make the calculations are readily obtained with standard modal techniques.				
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THE DIRECT MEASUREMENT OF DAMPING COEFFICIENTS USING AN INVERSE EIGENVALUE METHOD

1. INTRODUCTION

The measurement of system mass, damping, and stiffness coefficients is a classical engineering problem that has been addressed by a number of researchers (Beck and Arnold, 1977; Tomlinson, 1979; Fritzen, 1986; Wang, 1988; Lin and Plunkett, 1989; Cobb and Mitchell, 1990; Wang and Liou, 1990; Lee and Dobson, 1991). Their methods typically consist of measuring a frequency response function (transfer function) of a system and then curve fitting a multidegree of freedom model to the magnitude and phase angle of the system response. During this process, the system parameters are identified. The measurement of mass and stiffness coefficients is usually straightforward. The inclusion of damping into a system produces a bounded complex system response, and thus the extraction of these damping characteristics is more complicated. Since the damping coefficients are usually small in magnitude compared with system stiffness and mass parameters, the estimation (or measurement) of the damping coefficients has been frequently inaccurate.

This report develops a direct measurement technique to obtain the damping coefficient of a viscous damper. Using only a bar and a viscous absorber, this unique experiment will measure a viscous damping parameter that is frequency dependent. The complex-valued natural frequencies of this system are easily identified using a standard modal test. Unlike previous methods that rely on curve-fit techniques, the form of the measured eigenvalues can be inverted at each natural frequency to yield a direct measurement of viscous damping coefficient. An experiment is included to demonstrate the method.

2. SYSTEM MODEL

The system model consists of a one-dimensional axial bar, free at $x = 0$, and a viscous damper at $x = L$ (figure 1). A force is applied to the bar at location $x = 0$. The addition of the damper to the bar results in a bounded complex system transfer function (frequency response function, or FRF). The linear second-order wave equation modeling particle displacement in the bar is

$$\frac{\partial^2 u(x,t)}{\partial t^2} - s^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\delta(x)F(t)}{\rho A} , \quad (1)$$

where $u(x,t)$ is the particle displacement (m), ρ is the density of the bar (kg/m^3), s is the longitudinal wave speed in the bar (m/s), x is the spatial location (m), t is the time (s), A is the area of the bar (m^2), F is the applied force (N), and δ is the Dirac delta function (m^{-1}). The wave speed s is equal to the quantity of the modulus of elasticity E (N/m^2) divided by the density ($s = \sqrt{E/\rho}$). The wave equation assumes a uniform area and negligible internal loss in the bar.

The free boundary at $x = 0$ can be modeled as

$$\frac{\partial u}{\partial x}(0,t) = 0 . \quad (2)$$

The boundary condition at $x = L$ is obtained by matching the force at the end of the bar to the viscous dissipative force in the damper. This expression is

$$AE \frac{\partial u}{\partial x}(L,t) = -c \frac{\partial u}{\partial t}(L,t) , \quad (3)$$

where c is the viscous damping coefficient (Ns/m). When c is equal to zero (or infinity), the boundary at $x = L$ reflects all the wave energy, and the system response is composed only of standing waves. When c is equal to $A\sqrt{\rho E}$, the boundary at $x = L$ absorbs all the wave energy, and the system response is composed only of propagating waves. All other values of c exhibit some combination of standing and propagating wave energy in their response.

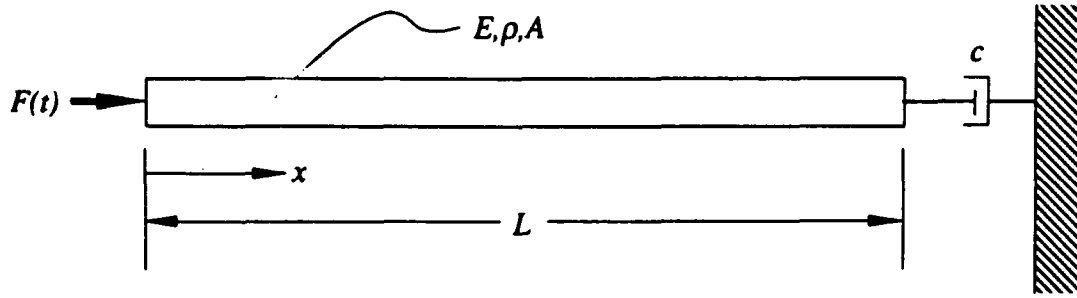


Figure 1. Beam With Viscous Damper

3. SEPARATION OF VARIABLES

The eigenvalues of the model are found by applying separation of variables to the homogeneous version of equation (1) and then to the boundary conditions in equations (2) and (3). Separation of variables assumes that the solution is a product of a function in the spatial domain multiplied by a function in the time domain:

$$u(x,t) = X(x)T(t) . \quad (4)$$

Inserting equation (4) into the homogeneous version of equation (1) produces two independent ordinary differential equations, each with the complex-valued separation constant λ : namely,

$$\frac{d^2 X(x)}{dx^2} - \lambda^2 X(x) = 0 \quad (5)$$

and

$$\frac{d^2 T(t)}{dt^2} - s^2 \lambda^2 T(t) = 0 . \quad (6)$$

The spatial ordinary differential equation given in equation (5) is solved using the boundary condition of equation (2):

$$X(x) = e^{\lambda x} + e^{-\lambda x} . \quad (7)$$

The time-dependent ordinary differential equation yields the following general solution:

$$T(t) = G e^{s\lambda t} + H e^{-s\lambda t} . \quad (8)$$

Applying the boundary condition of equation (3) to equations (5) and (6) yields $H = 0$ and the separation constants

$$\lambda_n = \frac{1}{2L} \log_e \left[\frac{AE - cs}{AE + cs} \right] + \frac{n\pi}{L} i \quad n = 0, \pm 1, \pm 2, \dots, \quad (9)$$

and

$$\lambda = 0. \quad (10)$$

The system eigenvalues Λ_n are equal to the separation constant multiplied by the wave speed s ($\Lambda_n = s\lambda_n$). An eigenvalue plot is shown in figure 2. Each of these eigenvalues is a function of the damping at the boundary. When the value of damping at the boundary is increased, the eigenvalues will move to the left in the complex plane. Critical damping for this system occurs when $AE = cs$. The inverse function of equation (9) allows the damping to be computed from the measured eigenvalues. Although equation (10), which represents a rigid body mode, is not used in the computation of damping, it must be used when the theoretical transfer function is computed.

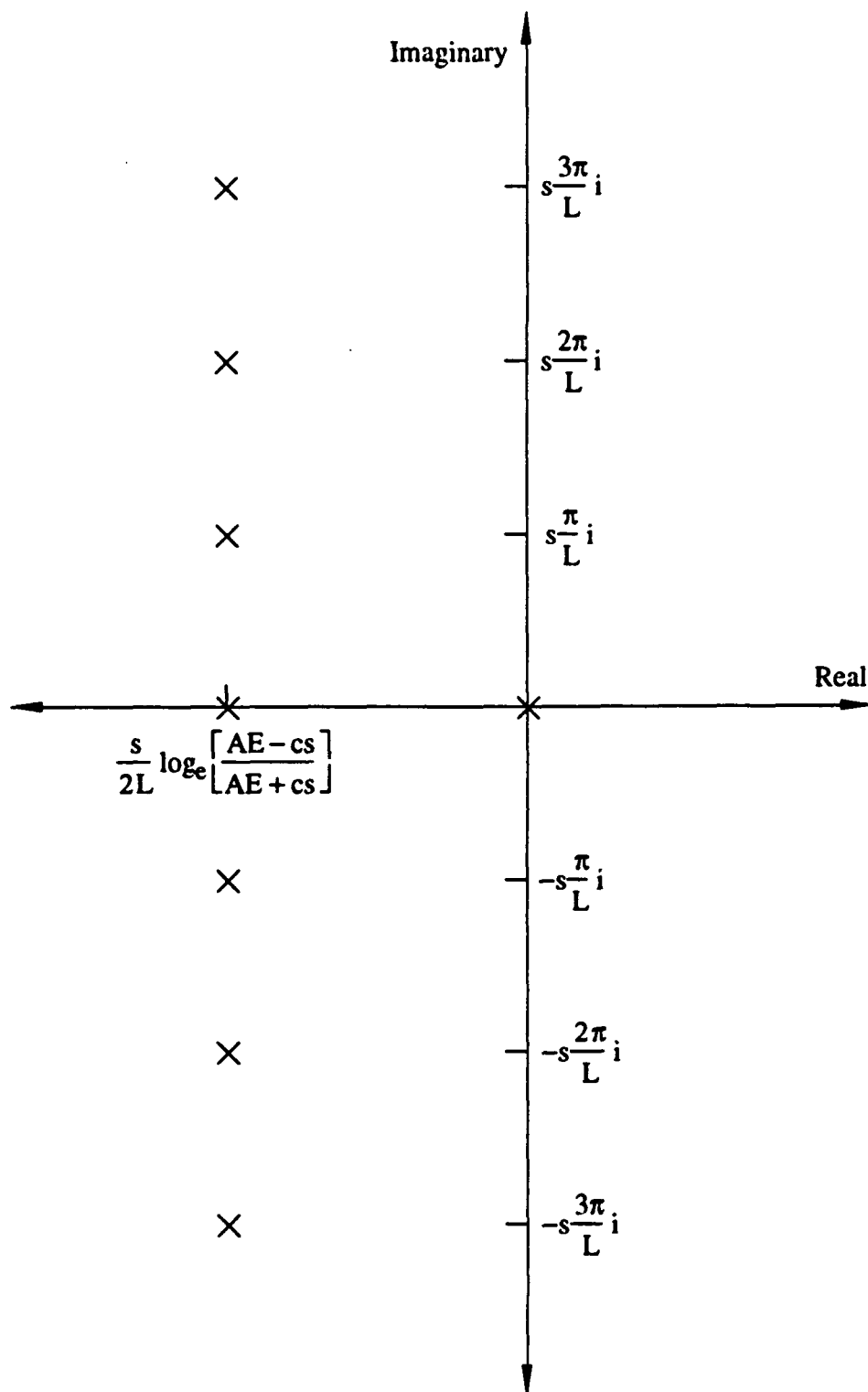


Figure 2. Eigenvalue Location in the Complex Plane

4. DAMPING COMPUTATION

The frequency-dependent damping coefficient c at $x = L$ can be determined at each bar resonance from the imaginary component of the eigenvalue at that resonance. The real and imaginary components of the eigenvalues are easily extracted from the measured transfer function, which is the bar response divided by the input force. The computation of damping coefficients begins by multiplying equation (9) by the wave speed, s , and is expressed as

$$\text{Re}(\Lambda_n) + i \text{Im}(\Lambda_n) = \frac{s}{2L} \log_e \left[\frac{AE - cs}{AE + cs} \right] + s \frac{n\pi}{L} i, \quad (11)$$

where $\text{Re}(\)$ denotes the real part, $\text{Im}(\)$ denotes the imaginary part, and the subscript n denotes the n th resonance. The real-valued terms from equation (11) can be written separately as

$$\text{Re}(\Lambda_n) = \frac{s}{2L} \log_e \left[\frac{AE - cs}{AE + cs} \right]. \quad (12)$$

Multiplying both sides by $2L/s$ and then taking the exponential of both sides to remove the natural log on the right-hand-side gives

$$\left[\frac{AE - cs}{AE + cs} \right] = \exp \left[\frac{2L}{s} \text{Re}(\Lambda_n) \right]. \quad (13)$$

Solving for c in equation (13) yields

$$c = \frac{AE}{s} \left\{ \frac{1 - \exp \left[\frac{2L}{s} \text{Re}(\Lambda_n) \right]}{1 + \exp \left[\frac{2L}{s} \text{Re}(\Lambda_n) \right]} \right\}, \quad (14)$$

where c is in units of Ns/m. The frequency-dependent damping coefficients of the viscous damper can be computed using equation (14) when the eigenvalues at the system resonance are known. Each damping value corresponds to the measured resonant frequency of the bar. Theoretically, when $AE = cs$, the system is critically damped, and the real parts of the eigenvalues are located at negative infinity in the complex plane. Experimentally, this very

large damping value is difficult to produce. If this limit could actually be reached, an increase in the bar area would result in a larger critical damping coefficient, which, in turn, would shift the real part of the eigenvalue locations of the system from negative infinity to discrete values. Thus, meaningful calculations of damping values from the measured system eigenvalues would be insured.

5. EXPERIMENT

The experimental apparatus consisted of a 6.096-m (20-ft) steel bar attached to a Monroe automotive shock absorber. The bar had a width and height of 0.0254 m (1 inch), which resulted in a cross-sectional area of 0.000645 m^2 (1 inch²). The shock absorber, a type 33116 PCB26J2E, was tested at the standard installed operational length of 4.57 m (18 inches). The end at $x = 0$ was excited with a Bruel and Kjaer (B&K) Type 8202 modal impact hammer containing a B&K Type 8200 force transducer. The bar response was measured at five locations using a B&K Type 4368 accelerometer. The two signals were input into a Hewlett-Packard 3562 dual channel spectrum analyzer that calculated the system frequency response function. This response function used the accelerometer as the output and the applied force as the input. The analyzer also evaluated the eigenvalues of the response function. The real component of the eigenvalues was used in the above equations to determine the damping in the shock absorber.

Table 1 shows the mean and standard deviations of the measured eigenvalues for the system. These values were calculated from five sets of measurements at five different locations ($x = 1.83, 2.13, 2.74, 3.66, \text{ and } 4.57 \text{ m}$ (6, 7, 9, 12, and 15 ft)). Each individual eigenvalue was measured from a transfer function composed of five averaged Fast Fourier transforms. The calculated damping values for the system at the natural frequencies are shown in table 2. The standard deviations of the damping coefficient at each measured frequency were 4.4, 15.4, and 12.8 percent for the first, second, and third resonances, respectively. These minimal deviations indicate that this technique provides a relatively stable measurement process for the dynamic viscous damping of a device.

The system transfer function can be computed using a modal method (Hull, to appear 1993), that assumes a harmonic force input at $x = 0$. This theoretical transfer function is

$$\frac{\ddot{u}(x, \omega)}{F_0} = \frac{-\omega^2}{2\rho AL} \sum_{n=-\infty}^{\infty} \frac{\varphi_n(x)}{(i\omega - \Lambda_n)\Lambda_n} - \frac{i\omega}{\rho AL} \sum_{n=-\infty}^{\infty} \frac{1}{\Lambda_n}, \quad (15)$$

where ω is the frequency (rad/s), i is the square root of -1, and the eigenfunctions $\varphi_n(x)$, given in equation (7), are evaluated using the n -indexed separation constant as

$$\varphi_n(x) = e^{\lambda_n x} + e^{-\lambda_n x}. \quad (16)$$

A comparison of equation (15) to the experiment is shown in figure 3. The solid line is the theoretical transfer function and the dashed line is the experiment at $x = 2.13$ m (7 ft). Here, equation (15) was evaluated using seven terms ($-3 \leq n \leq 3$), where n is an integer). The transfer function was evaluated by inserting the calculated damping value at each resonance into the theoretical eigenvalue. Due to the symmetry of the problem, the damping values obtained for the positive n modes were used for the corresponding negative n modes. The damping at the $n = 0$ mode was evaluated using the value obtained from the $n = 1$ mode. This process allowed the frequency-dependent characteristic of the damping to be incorporated into the theoretical transfer function. The 7.2-percent average difference between the theory and the measurement was calculated using the equation

$$\% \text{ difference} = \left| \frac{\text{Theory} - \text{Experiment}}{\text{Max(Theory)}} \right| \cdot 100. \quad (17)$$

This equation allows percentage differences to be calculated while effectively ignoring the nulls of the system transfer function. It should not be used for systems with very low damping because the large magnitudes present will tend to distort differences between data and theory.

Table 3 compares the theoretical imaginary natural frequency to the experimentally measured natural frequency. The percentage differences in this table were calculated using

$$\% \text{ difference} = \left| \frac{\text{Theory} - \text{Experiment}}{\text{Theory}} \right| \cdot 100 . \quad (18)$$

The comparison between theory and experiment shows very good agreement. The slight difference between the theoretical and the experimental results is due to the stiffness contribution of the shock absorber to the system. The small percentage difference in table 3 ensures that the stiffness of the bar is much greater than the stiffness of the damping device, which is required if the damper is to be modeled as a pure loss term.

Table 1. Measured System Mean Eigenvalues and Standard Deviations

Eigenvalue (<i>n</i>)	Re(Λ_n) mean, μ	Re(Λ_n) std. dev., σ	Im(Λ_n) mean, μ (Hz)	Im(Λ_n) std. dev., σ (Hz)
1	-5.65	0.25	401.	0.311
2	-20.9	3.21	848.	5.53
3	-21.0	2.68	1230.	17.2

Table 2. Computed System Damping

Eigenvalue (<i>n</i>)	Frequency (Hz)	Damping, <i>c</i> (Ns/m)	Damping Ratio, ζ
1	401.	1090.	0.042
2	848.	4000.	0.154
3	1230.	4020.	0.155

Table 3. Comparison of Theoretical to Experimental Imaginary Natural Frequencies

Eigenvalue (<i>n</i>)	Theoretical Frequency (Hz)	Experimental Frequency (Hz)	Percent Difference %
1	422.	401.	5.0
2	845.	848.	0.4
3	1267.	1230.	2.9

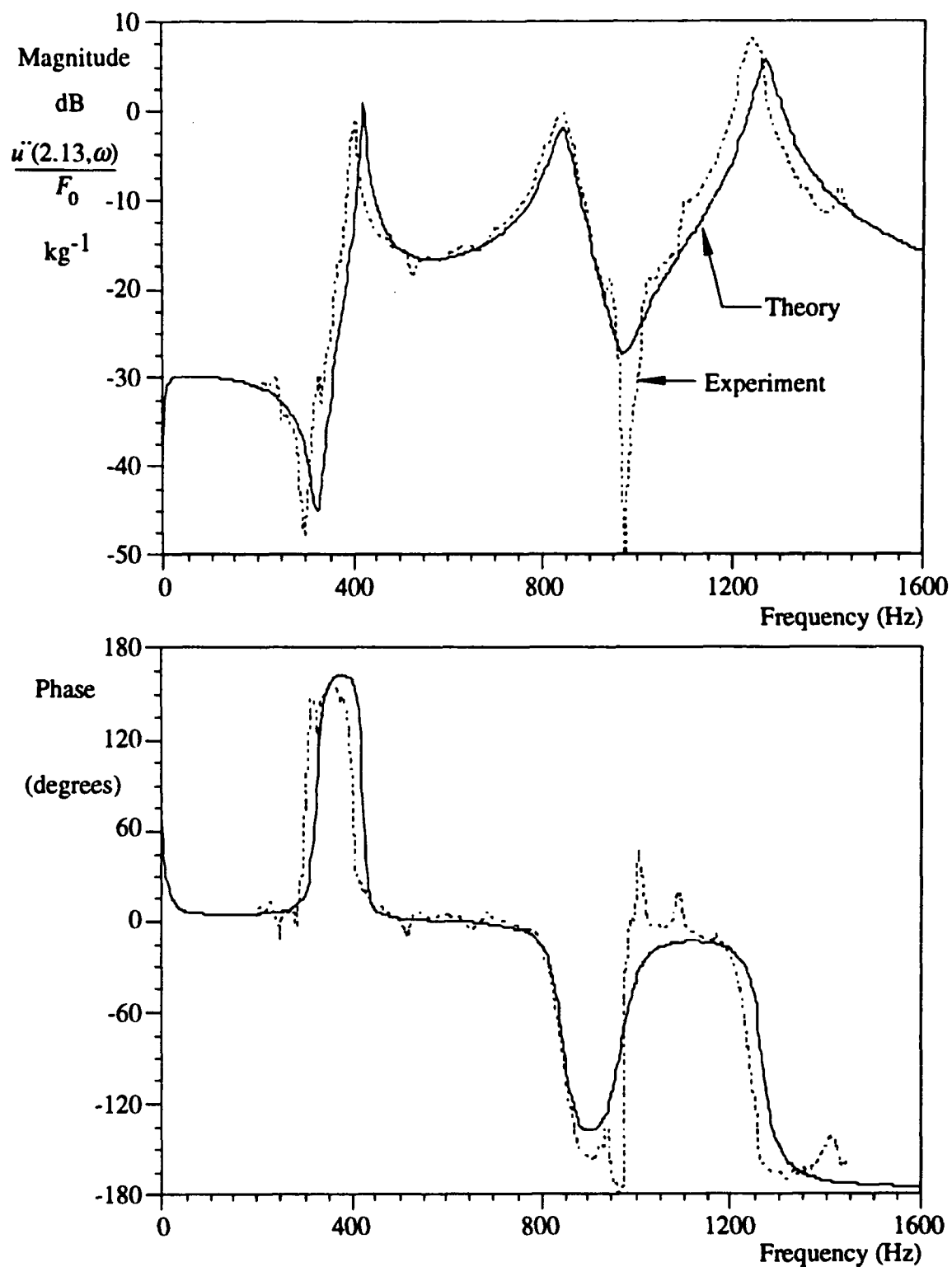


Figure 3. Transfer Function of Acceleration Response Divided by Force Input Versus Frequency at $x = 2.13$ Meters

6. CONCLUSIONS

A new method has been developed to measure frequency-dependent damping coefficients of one-dimensional devices. This direct measurement technique relies on the theoretical formulation of the eigenfunctions and eigenvalues of a longitudinal bar with free-end and damped-end boundary conditions. These eigenvalues are complex functions dependent on the damping coefficient at the boundary, and on the length, density, elastic modulus, and cross-sectional area of the bar. The functional form of the eigenvalues can be inverted such that the damping coefficient at the boundary becomes a function of the real part of the eigenvalue and beam properties of the system. Such inversion is useful because the damping coefficient is now a function of measured quantities. The well-known properties of the bar can be varied through design, and the eigenvalues can be extracted from a frequency response function (transfer function) of the system. A simple experiment was developed to measure this function using modal impact techniques. The damping at the boundary was computed by a closed form solution after the eigenvalues of the experimental transfer function had been identified. The results of this experiment show that the standard deviation of the damping coefficient at each measurement frequency ranged from 4.5 to 15.4 percent. The average deviation between the magnitude of the measured transfer function and the magnitude of the theoretical transfer function was 7.2 percent. Such a small deviation indicates a very stable measurement technique.

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